Dielectric

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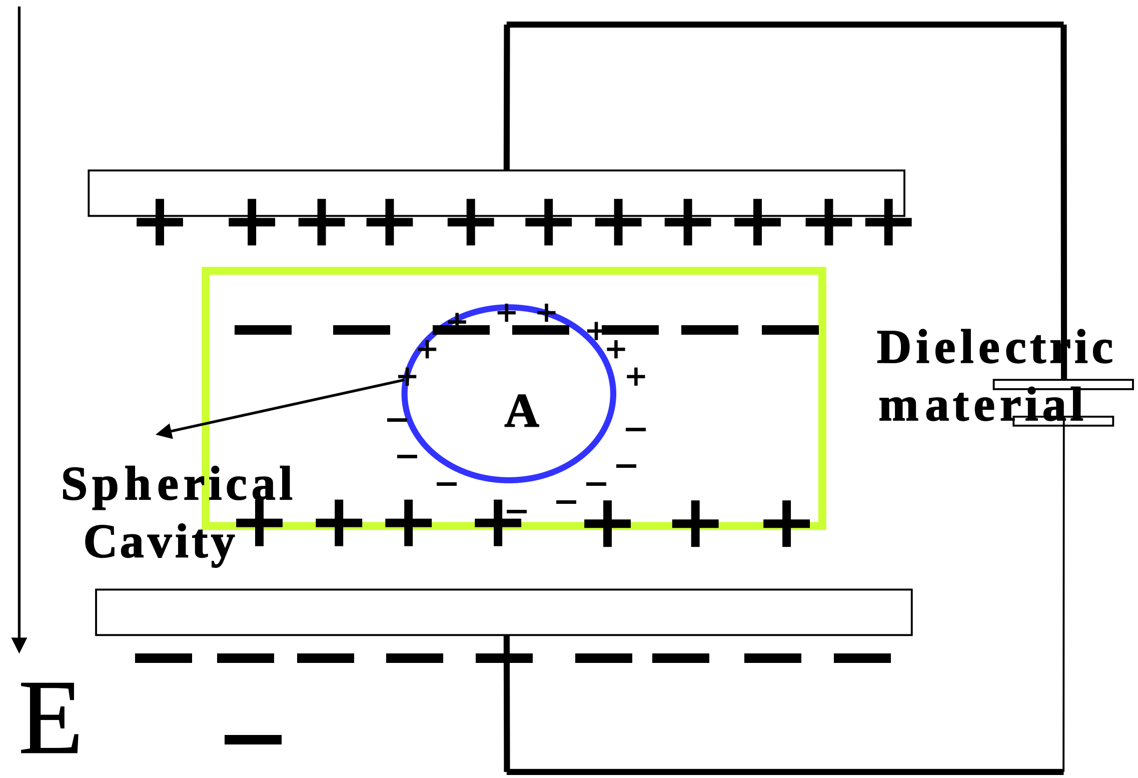
Dielectric

Internal fields or local fields

Local field or internal field in a dielectric is the space and time average of the electric field intensity acting on a particular molecule in the dielectric material.



Dielectric

Field E_1

E_1 is the field intensity at A due to the charge density on the plates

$$E_1 = \frac{D}{\varepsilon_0}$$

$$D = \varepsilon_0 E + P$$

$$E_1 = \frac{\varepsilon_0 E + P}{\varepsilon_0}$$

$$E_1 = E + \frac{P}{\varepsilon_0} \ldots\ldots\ldots\ldots(1)$$

Dielectric

Field E_2

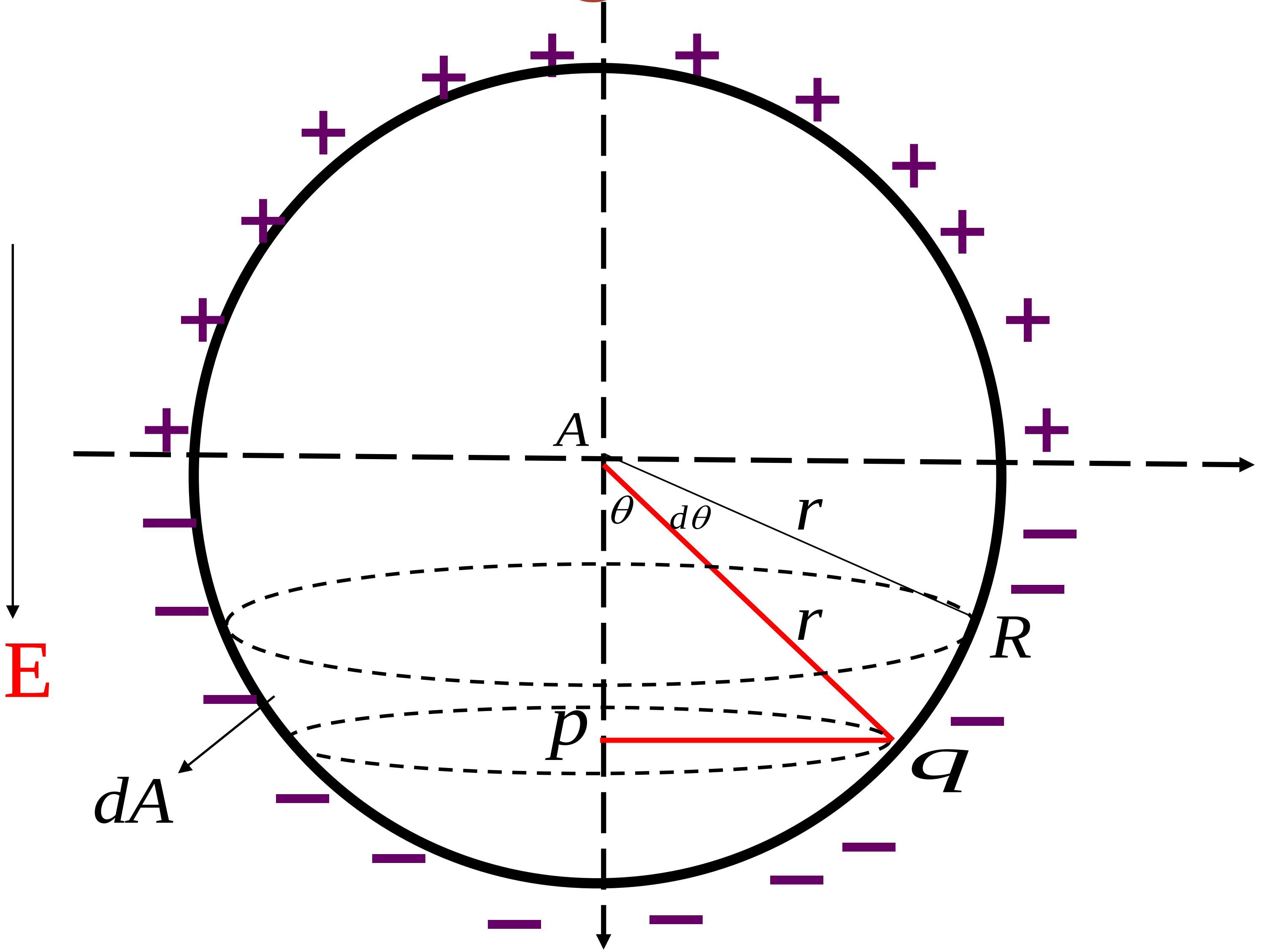
E_2 is the field intensity at A due to the charge density induced on the two sides of the dielectric.

$$E_2 = \frac{-P}{\varepsilon_0} \qquad \ldots\ldots\ldots\ldots(2)$$

Field E_3

E_3 is the field intensity at A due to the atoms contained in the cavity, we are assuming a cubic structure, so $E_3 = 0$.

Dielectric



Dielectric

Field E_4:

This is due to polarized charges on the surface of the spherical cavity.

$$dA = 2\pi . pq.qR$$
$$dA = 2\pi . r \sin\theta . rd\theta$$
$$dA = 2\pi . r^2 \sin\theta d\theta$$

Where dA is Surface area between θ & θ+dθ...

2.The total charge present on the surface area dA is...

dq = (normal component of polarization) X (surface area)

$$dq = p\cos\theta \times dA$$
$$dq = 2\pi r^2 p\cos\theta . \sin\theta . d\theta$$

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3.The field due to this charge at A, denoted by dE_4 is given by

$$dE_4 = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}$$

The field in θ = 0 direction

$$dE_4 = \frac{1}{4\pi\varepsilon_0}\frac{dq\cos\theta}{r^2}$$

$$dE_4 = \frac{1}{4\pi\varepsilon_0 r^2}(2\pi r^2 p\cos\theta.\sin\theta.d\theta)\cos\theta$$

$$dE_4 = \frac{P}{2\varepsilon_0}\cos^2\theta.\sin\theta.d\theta$$

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4.Thus the total field E_4 due to the charges on the surface of the entire cavity is

$$E_4 = \int_0^{\pi} dE_4$$

$$= \int_0^{\pi} \frac{P}{2\varepsilon_0} \cos^2\theta . \sin\theta . d\theta$$

$$= \frac{P}{2\varepsilon_0} \int_0^{\pi} \cos^2\theta . \sin\theta . d\theta$$

$$let..x = \cos\theta \rightarrow dx = -\sin\theta d\theta$$

$$= \frac{P}{2\varepsilon_0} \int_1^{-1} x^2 . dx$$

$$= \frac{-P}{2\varepsilon_0} \left(\frac{x^3}{3}\right)_1^{-1} \Rightarrow \frac{-P}{2\varepsilon_0} \left(\frac{-1-1}{3}\right)$$

$$E_4 = \frac{P}{3\varepsilon_0}$$

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The internal field or Lorentz field can be written as

$$E_i = E_1 + E_2 + E_3 + E_4$$

$$E_i = (E + \frac{p}{\varepsilon_o}) - \frac{p}{\varepsilon_o} + 0 + \frac{p}{3\varepsilon_o}$$

$$E_i = E + \frac{p}{3\varepsilon_o}$$

Dielectric

Classius – Mosotti relation

Consider a dielectric material having cubic structure , and assume ionic Polarizability & Orientational polarizability are zero.

$$\alpha_i = \alpha_0 = 0$$

$$polarization..P = N\mu$$

$$P = N\alpha_e E_i ......where., \mu = \alpha_e E_i$$

$$where., E_i = E + \frac{P}{3\varepsilon_0}$$

Dielectric

$$P = N\alpha_e E_i$$

$$P = N\alpha_e\left(E + \frac{P}{3\varepsilon_0}\right)$$

$$P = N\alpha_e E + N\alpha_e \frac{P}{3\varepsilon_0}$$

$$P - N\alpha_e \frac{P}{3\varepsilon_0} = N\alpha_e E$$

$$P\left(1 - \frac{N\alpha_e}{3\varepsilon_0}\right) = N\alpha_e E$$

$$P = \frac{N\alpha_e E}{\left(1 - \frac{N\alpha_e}{3\varepsilon_0}\right)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Dielectric

We known that the polarization vector

$$P = \varepsilon_0 E(\varepsilon_r - 1)............(2)$$

from eq$^n s$(1) & (2)

$$\frac{N\alpha_e E}{(1 - \frac{N\alpha_e}{3\varepsilon_0})} = \varepsilon_0 E(\varepsilon_r - 1)$$

$$1 - \frac{N\alpha_e}{3\varepsilon_0} = \frac{N\alpha_e E}{\varepsilon_0 E(\varepsilon_r - 1)}$$

$$1 = \frac{N\alpha_e}{3\varepsilon_0} + \frac{N\alpha_e E}{\varepsilon_0 E(\varepsilon_r - 1)}$$

$$1 = \frac{N\alpha_e}{3\varepsilon_0} + \frac{N\alpha_e}{\varepsilon_0(\varepsilon_r - 1)}$$

$$1 = \frac{N\alpha_e}{3\varepsilon_0}(1 + \frac{3}{\varepsilon_r - 1})$$

$$\frac{N\alpha_e}{3\varepsilon_0} = \frac{1}{(1 + \frac{3}{\varepsilon_r - 1})}$$

$$\frac{N\alpha_e}{3\varepsilon_0} = \frac{\varepsilon_r - 1}{\varepsilon_r + 2} ...... \rightarrow \text{Classius Mosotti relation}$$